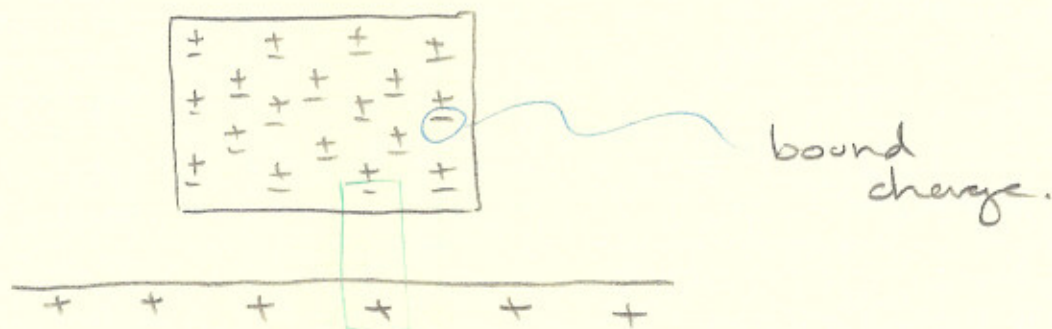


Free charge

$$\oint \vec{D} \cdot d\vec{S} = q_{\text{free}}$$

$$\vec{D} = \epsilon \vec{E}$$

already accounts for bound charges.

EX:

$$\epsilon_r = 2$$

Dielectric sphere of radius R

$$\epsilon = \epsilon_r \epsilon_0$$

Find \vec{E} inside if $\rho = \rho_0$

$$\oint \vec{D} \cdot d\vec{S} = \rho_0 \frac{4\pi}{3} r^3$$

free charges.

when $r_p < R$

$$|\vec{D}(r_p)| \cdot 4\pi r_p^2$$

$$|\vec{D}(r_p)| \cdot r = \rho_0 \cdot \frac{4\pi}{3} r_p^3 \cdot \frac{1}{4\pi r_p^2}$$

$$= \frac{\rho_0}{3} r_p$$

$$|\vec{D}| = \epsilon |\vec{E}|$$

$$|\vec{E}| = \frac{\rho_0 r_p}{3\epsilon_0}$$

$$= \frac{\rho r_p}{6\epsilon_0}$$

$$\underline{r_p > R}$$

$$\oint \vec{D} \cdot d\vec{S} = q_{free}$$

$$|\vec{D}(r_p)| 4\pi r_p^2 = \rho_0 \frac{4}{3} \pi R^3$$

Problem: Which ϵ do we use?

We use the ϵ of the vacuum b/c that is where we are.

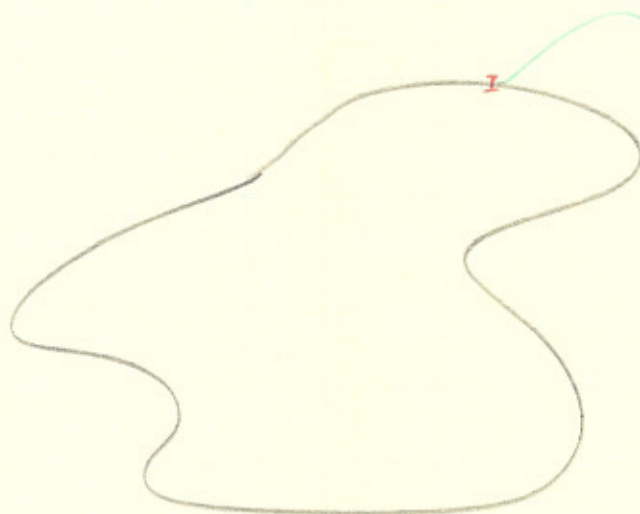
Note: $\epsilon_r(\text{air}) = 1.0007 \approx \epsilon_r(\text{vacuum})$

If there is more than one medium w/ diff. ϵ , then we need to study what happens at the boundary between the media.

Electric Boundary conditions. (static case)

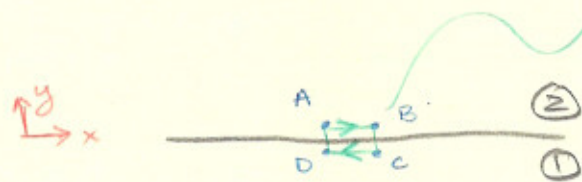
$$\oint \vec{E} \cdot d\vec{l} = 0$$

$$\oint \vec{D} \cdot d\vec{S} = q_{\text{free.}}$$



What happens
when when
go from
inside the
material to
outside?

We look at peices that are so small they are basically flat.



this is a
very small
circuit.

$$\oint \vec{E} \cdot d\vec{l} = 0$$

$$= \int_A^B \vec{E} \cdot d\vec{l} + \int_B^C \vec{E} \cdot d\vec{l} + \int_C^D \vec{E} \cdot d\vec{l} + \int_D^A \vec{E} \cdot d\vec{l}$$

$$\vec{E} = E_x \hat{x} + E_y \hat{y} + E_z \hat{z}$$

$$\int_A^B \vec{E} \cdot d\vec{l}$$

$$d\vec{l} = \hat{x} dx$$

$$= E_x dx = |\vec{E}| \Delta l$$

$$\int_C^B \vec{E} \cdot d\vec{l} = |\vec{E}| \Delta l$$

we make so small it
is equal to zero.
negligible

$$\int_C^D \vec{E} \cdot d\vec{l} = -|\vec{E}| \cdot \Delta l$$

$$\int_D^A \sim \Delta h \rightarrow 0$$

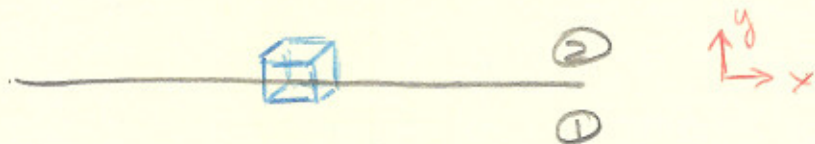
$$\oint \vec{E} \cdot d\vec{l} = (|E_{x1}| - |E_{x2}|) \Delta l = 0$$

$$\therefore |E_{x1}| = |E_{x2}|$$

This E_x is the component that is to the surface

Looking at the other condition

$$\oint \vec{D} \cdot d\vec{S} = q_{\text{free}}$$



We make the height of our box so small that the flux lost through the side panels is negligible.

$$\therefore \oint \vec{D} \cdot d\vec{S} = \int_{\text{Top}} \vec{D} \cdot d\vec{S} + \int_{\text{Bottom}} \vec{D} \cdot d\vec{S} + \int_{\text{sides}} \vec{D} \cdot d\vec{S}$$

$$(D_{y②}) \Delta S - (D_{y①}) \Delta S = \sigma_{\text{free}} \Delta S$$

$$D_{y②} \Rightarrow D_{n,z}$$

$D_{n,z}$ = component of \vec{D} that is normal to the interface.

What is $\sigma_{\text{free}} = \sigma_{\text{free},z} + \sigma_{\text{free},1}$

$$(D_{n,z} - D_{n,z}) = \sigma_{\text{free}}$$

$$\epsilon_1 E_{n,z} - \epsilon_2 E_{n,z} = \sigma_{\text{free}}$$